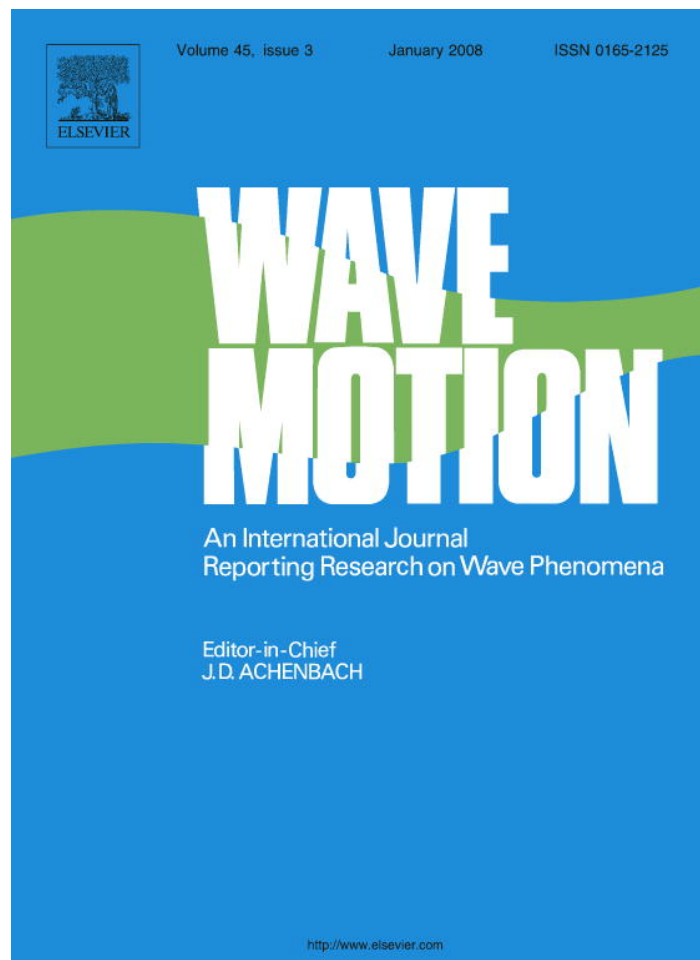


Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.



This article was published in an Elsevier journal. The attached copy is furnished to the author for non-commercial research and education use, including for instruction at the author's institution, sharing with colleagues and providing to institution administration.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



# Influence of viscosity on the reflection and transmission of an acoustic wave by a periodic array of screens: The general 3-D problem

Dorel Homentcovschi <sup>\*</sup>, Ronald N. Miles

*Department of Mechanical Engineering, SUNY Binghamton, NY 13902-6000, USA*

Received 28 November 2006; received in revised form 25 May 2007; accepted 29 May 2007

Available online 14 June 2007

---

## Abstract

An analysis is presented of the diffraction of a pressure wave by a periodic grating including the influence of the air viscosity. The direction of the incoming pressure wave is arbitrary. As opposed to the classical nonviscous case, the problem cannot be reduced to a plane problem having a definite 3-D character. The system of partial differential equations used for solving the problem consists of the compressible Navier–Stokes equations associated with no-slip boundary conditions on solid surfaces. The problem is reduced to a system of two hypersingular integral equations for determining the velocity components in the slits' plane and a hypersingular integral equation for the normal component of velocity. These equations are solved by using Galerkin's method with some special trial functions. The results can be applied in designing protective screens for miniature microphones realized in MEMS technology. In this case, the physical dimensions of the device are on the order of the viscous boundary layer so that the viscosity cannot be neglected. The analysis indicates that the openings in the screen should be on the order of 10  $\mu\text{m}$  in order to avoid excessive attenuation of the signal. This paper also provides the variation of the transmission coefficient with frequency in the acoustical domain.

© 2007 Elsevier B.V. All rights reserved.

*Keywords:* Acoustic wave; Viscosity; Diffraction; Grating; MEMS

---

## 1. Introduction

The problem of diffraction of a plane wave by a periodic plane grating consisting of thin (zero thickness), coplanar, equally spaced barriers with parallel edges is a classical problem of wave propagation. The problem admits acoustic, electromagnetic, elastic solid, and water wave interpretations and has been studied by many authors. For the normal incidence case, Lamb [1] obtained analytical formulas for the reflection and transmission coefficients in the low frequency limit. Miles [2] studied the case of a grating of inclined flat plates in a one-mode approximation for small screens. Achenbach and Li [3] have developed an exact method suitable

---

<sup>\*</sup> Corresponding author. Permanent address: *Politechnica* University of Bucharest, Applied Science Department, Bucharest, Romania. Tel.: +1 607 777 5008; fax: +1 607 777 4620.

*E-mail address:* [homentco@binghamton.edu](mailto:homentco@binghamton.edu) (D. Homentcovschi).

for arbitrary frequency and incidence. They utilized a periodic Green's function to reduce the problem to a singular integral equation. This equation has been solved by expanding the unknown function in Chebyshev polynomials and solving for the coefficients from a set of algebraic equations resulting from a Galerkin-type approach. Finally, we note the paper of Scarpetta and Sumbatyan [4] which provides some explicit analytical formulas for reflection and transmission coefficients in the one-mode oblique incidence case.

Recent progress in integrated circuit technology has enabled the fabrication of microelectromechanical systems (MEMS) including accelerometers, pressure sensors, etc. The development has led toward small (subminiature) microphones with diaphragms of order of 1 mm. In order to protect the mobile part of the device from dust particles and water drops some protecting structures have been designed. A simple model of such a protecting screen is a periodic plane grating. It is clear that the fluid viscosity must have an increasingly important effect on sound transmission through a grating as the hole sizes decrease. In fact, it can be shown that the boundary layer associated with the plane wave air motion past a hard plane is of the order of 1 mm, the same as the order of the dimensions of the devices.

The influence of viscosity on the pressure wave propagation was considered in only a few papers. In 1957 Alblas [5] solved the problem of scattering of a pressure wave by a halfplane considering the viscosity of the air. Pierce's book [6] contains a chapter dedicated to effects of viscosity and other dissipative processes. In the paper [7] the 2-D propagation of a damped acoustical wave through a periodic grating was considered. This paper analyzed the particular case when the incoming plane wave has its propagation direction perpendicular to the gratings' strips direction (referred henceforth as the longitudinal direction). Hence, the analysis in the present paper is a direct generalization of the work in [7].

The present paper gives a complete solution for the linear boundary-value problem corresponding to the reflection and transmission of an incident, time-harmonic plane wave by a periodic system of screens for a viscous fluid. We start in Section 2 with the linearized equations describing the motion of a viscous fluid in the absence of a mean flow; the corresponding nonslip boundary condition on the plane screen and also Sommerfeld's radiation condition complete the formulation of the boundary-value problem.

Due to the special geometry of the problem (translation invariance parallel to the longitudinal  $y$ -axis) as with the nonviscous case, a reduced problem (independent of  $y$ -variable), can be formulated [8]. But, in contrast to the inviscid case, due to the continuity equation which involves all the velocity components, the problem cannot be treated any more as a 2-D problem corresponding to the case where the incoming wave is propagating perpendicular to the strips' direction.

In the next section, representation formulas for pressure and velocity in the upper and lower half-spaces are given. These formulas result by using Fourier Transforms of the linearized continuity and momentum equations, with respect to  $x$  and  $y$  variables (by  $xOy$  we denote the plane containing the strips). The incoming wave is considered as a plane pressure wave characterized by unit vector  $\mathbf{n}_0$ . As the attenuation of the sound waves in air at atmospheric pressure is very small, we neglect it in all propagating modes (incoming wave and outgoing waves). This is why, despite the viscous dissipation, we continue to use Sommerfeld's condition. The incoming pressure wave is perturbed by the gratings' strips giving reflected and transmitted waves. The representation formulas for reflected and transmitted pressures contain an infinite number of wave modes, each with its own cut-off frequency. At the cut-off frequency, a mode converts from a propagating wave mode into an evanescent mode. At small frequencies only the lower order modes are propagating. As the frequency is increasing, more and more evanescent modes convert to propagating modes. For audible frequencies in air, which is the case we are considering in this paper, only the lowest modes are propagating. The case when also other modes are propagating can be analyzed similarly. The velocity field contains besides the propagating modes, generated by the pressure waves, a system of viscous (vorticity) modes which are decaying exponentially with the distance from perturbation sources.

The values of velocity components along the slits are taken as the main unknown functions. All the coefficients entering in representation formulas are expressed in terms of Fourier Transforms of velocity components. We impose also the natural conditions of continuity of the velocity and its normal derivative across the slits; these conditions give the functional equations of the problem. There results a system of two equations connecting the plane components of velocity and, a separate integral equation corresponding to the  $z$ -component of velocity. The functional equations contain divergent series and, consequently, they can be understood properly only within distributions theory. We succeeded in transforming the functional equations into hypersingular integral equations.

In Section 5 is given a method for solving the hypersingular integral equations by considering a basis of Chebyshev-type functions and taking advantage of the convenient form of the convolution product in the spectral domain. By a Galerkin technique we obtain infinite systems of linear (algebraic) equations for determining the expansion coefficients of the velocity components in the considered basis. The coefficients of linear systems result by using spectral properties of the singular part of integral equations, FFT transform of some smooth functions and the summation of some convergent infinite series. Since the chosen Chebyshev type bases describe quite well the singular behavior of the velocity derivatives at the ends of the slits the final systems are well conditioned and only a few terms are sufficient for obtaining a good approximation of the velocity components.

Results are included in Section 6. A comparison is made of the transmission coefficient for the case of non-viscous and viscous fluid. The solution for the nonviscous (classical) case is determined by using the computational technique developed in this paper. The obtained results coincide very well with the values resulting from the other approaches of the inviscid problem (for example, [4]). Also, in the case where the incident wave is propagating perpendicular to the strips' direction the results obtained by using this approach coincide with those of the 2-D case given in [7].

The results for the viscous case show a strong attenuation of the waves in the case where the slit width is small (less than 10  $\mu\text{m}$ ). In this paper, we also analyze the influence of the incoming wave direction upon the transmission coefficient. The transmission coefficient is practically insensitive with respect to the azimuthal coordinate. The paper also gives some graphs showing the variation of the transmission coefficient with frequency in the audible frequency range. The obtained data can be used for designing protecting screens for miniature microphones.

At last but not least we note that the consideration of viscosity enabled us to avoid the nonphysical singularity of the classical (nonviscous) solution at the strips' edges [5].

## 2. The equations of the problem

### 2.1. Formulation of the problem

We consider the penetration of a pressure wave through the array of coplanar rigid screens located at  $z = 0$  in Fig. 1a. The screens are infinitely long in the  $y$ -direction, the opening between two neighboring screens is  $2a$  and the period of the grating is  $T = a + b$  (Fig. 1b). We denote by  $\mathcal{D}^+$  the upper half-plane ( $z > 0$ ) and by  $\mathcal{D}^-$  the half plane  $z < 0$ . The incident wave is located in the domain  $\mathcal{D}^+$  and its propagation vector is given by the unit vector  $\mathbf{n}_0$ .

There are two periodic phenomena in this problem: one is associated with the acoustical incoming wave and the other one with the grating periodicity. To avoid possible confusions we associate a “\*” to the quantities

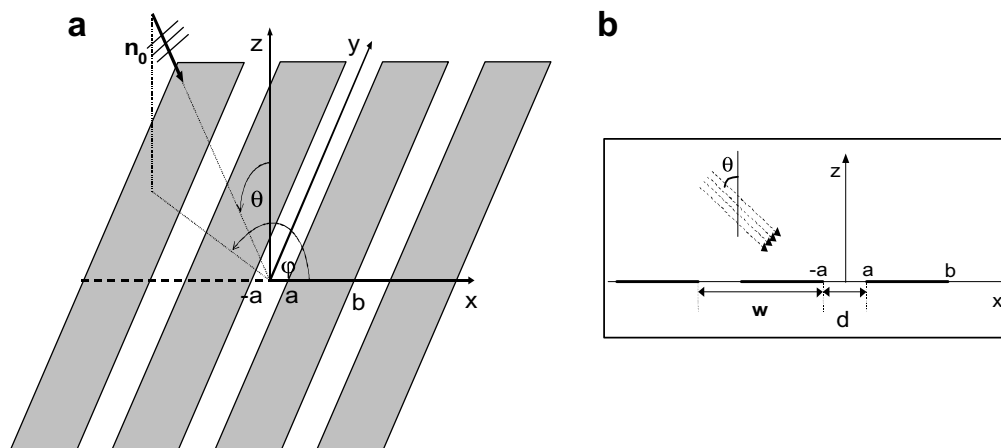


Fig. 1. The geometry of the problem: (a) the 3-D view, (b) a 2-D view.

related to the acoustical incoming wave ( $k^*$  is the angular frequency wave number of the plane incoming wave and  $\omega^*$  its angular frequency). The “nonstared” quantities  $T$  and  $\omega = 2\pi/T$  are the spatial period of the grating and its corresponding spatial frequency, respectively. The notations we are using are consistent with those in paper [7].

2.2. *The equations of the motion of a viscous fluid in linear acoustic approximation*

In the case where the coordinate system is chosen so that the unperturbed fluid is at rest (having the density  $\rho_0$  and the isentropic sound velocity  $c_0$ ) the first-order equations describing the motion of the gas can be written as [6,7,8]

$$\frac{1}{c_0^2} \frac{\partial p'}{\partial t} + \nabla \cdot \mathbf{v}' = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}'}{\partial t} + \nabla \left[ \frac{p'}{\rho_0} - (v' - v) \nabla \cdot \mathbf{v}' \right] - \nu \Delta \mathbf{v}' = 0 \tag{2}$$

where  $p'$  and  $\mathbf{v}'$  denote the pressure and velocity perturbations and

$$v = \frac{\mu}{\rho_0}, \quad v' = \frac{\mu_B}{\rho_0} + \frac{4\mu}{3\rho_0}$$

are the kinematic viscosities. Also,  $\mu$  and  $\mu_B$  are the shear and bulk viscosities [6].

We consider the case where all the field variables are harmonic in time with the same frequency  $\omega^* = 2\pi f$ . The case of general time dependence can be obtained, after analyzing each frequency separately, by Fourier superposition. In the case of simple harmonic oscillations in time we shall write

$$\{p'(\mathbf{x}, t), \mathbf{v}'(\mathbf{x}, t)\} = \{p(\mathbf{x}), \mathbf{v}(\mathbf{x})\} \exp(-i\omega^* t)$$

In this case the continuity Eq. (1) becomes

$$\nabla \cdot \mathbf{v} = \frac{i\omega^*}{c_0^2} \frac{p}{\rho_0} \tag{3}$$

Also, the momentum equation can be written as

$$\Delta \mathbf{v} + \frac{i\omega^*}{\nu} \mathbf{v} = \gamma \nabla \frac{p}{\rho_0} \tag{4}$$

where it was denoted

$$\gamma = \frac{1 + (v - v')i\omega^*/c_0^2}{\nu}$$

The relationships (3) and (4) give the equation for the pressure

$$[\Delta + k^{*2}] p = 0 \tag{5}$$

Here, we have used the notation

$$k^* = \frac{\omega^*}{\sqrt{c_0^2 - i\omega^* \nu}}, \quad \text{Im}(k^*) \geq 0 \tag{6}$$

Eqs. (4) and (5) constitute a system of four partial differential equations for determining the pressure and velocity fields.

For a viscous fluid we have the nonslip boundary condition

$$\mathbf{v}(\mathbf{x}) = \mathbf{0} \tag{7}$$

on any immobile solid surface.

Beside this, we will impose that all the propagating perturbations, except for the incoming plane wave, are outgoing waves (*Sommerfeld radiation condition*).

### 3. The representation formulas for the pressure and velocity fields

Let us consider an incoming plane pressure wave in  $\mathcal{D}^+$

$$p_{\omega}^{(\text{in})}(\mathbf{x})/\rho_0 = A \exp\{ik^* \mathbf{n}_0 \cdot \mathbf{x}\}$$

where  $\mathbf{n}_0 = n_{0x}\hat{\mathbf{x}} + n_{0y}\hat{\mathbf{y}} + n_{0z}\hat{\mathbf{z}}$  is the unit vector of the propagation direction of the wave. It can be verified directly that the function  $p_{\omega}^{(\text{in})}$  satisfies the pressure Eq. (5). Also, Eqs. (3) and (4) provide the corresponding velocity field as

$$\mathbf{v}_{\omega}^{(\text{in})}(\mathbf{x}) = i\delta k^* A \mathbf{n}_0 \exp\{ik^* \mathbf{n}_0 \cdot \mathbf{x}\}$$

where it has been denoted

$$\delta = \frac{1 + (v - v')i\omega^*/c_0^2}{i\omega^* - vk^{*2}}$$

The special geometry of the problem with respect to the  $y$ -axis and the form of the incoming wave suggest the determination of the solution in the form

$$p_{\omega}(\mathbf{x}) = p(x, z) \exp\{ik^* n_{0y}y\}$$

$$\mathbf{v}_{\omega}(\mathbf{x}) = \mathbf{v}(x, z) \exp\{ik^* n_{0y}y\}$$

The problem consisting of determining the functions  $p(x, z)$  and  $\mathbf{v}(x, z)$  will be referred as “*the reduced problem*”. Due to the continuity equation, which involves all the velocity components, the reduced problem differs from the 2-D case when the incident wave direction is contained in a plane perpendicular to the strips and containing the  $x$ -axis. This is also an important difference from the nonviscous case when the general 3-D problem can be solved by means of a 2-D problem (see Ref. [8]).

Eqs. (3) and (4) become

$$\frac{\partial u}{\partial x} + ik^* n_{0y}v + \frac{\partial w}{\partial z} = \frac{i\omega^*}{c_0^2} \frac{p}{\rho_0} \tag{8}$$

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - k^{*2} n_{0y}^2 + \frac{i\omega^*}{v} \right] \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \gamma \begin{bmatrix} \partial p / \partial x \\ ik^* n_{0y} p \\ \partial p / \partial z \end{bmatrix} \tag{9}$$

Now in order to use the periodicity of the grating with respect to  $x$ -axis we write

$$\mathbf{v}^{\pm}(x, z) = \exp(ik^* n_{0x}x) \tilde{\mathbf{v}}^{\pm}(x, z) \tag{10}$$

$\tilde{\mathbf{v}}^{\pm}(x, z)$  being a periodic function in  $x$

$$\tilde{\mathbf{v}}^{\pm}(x + T, z) = \tilde{\mathbf{v}}^{\pm}(x, z) \tag{11}$$

Therefore we obtain

$$\mathbf{v}^{\pm}(x, z) = \sum_{-\infty}^{+\infty} \tilde{\mathbf{v}}_n^{\pm}(z) \exp(ik_n x) \tag{12}$$

where  $\tilde{\mathbf{v}}_n^{\pm}(z) = (\tilde{u}_n^{\pm}(z), \tilde{v}_n^{\pm}(z), \tilde{w}_n^{\pm}(z))$  are the complex Fourier coefficients of the function  $\tilde{\mathbf{v}}^{\pm}(x, z)$

$$\tilde{\mathbf{v}}_n^{\pm}(z) = \frac{1}{T} \int_{-a}^{+a} \mathbf{v}^{\pm}(x', z) \exp(-in\omega x') dx' \tag{13}$$

It has been denoted

$$k_n = n\omega + k^* n_{0x}, \omega = 2\pi/T$$

The system of Eqs. (8) and (9) becomes

$$\frac{d\tilde{w}_n^\pm}{dz} + ik_n\tilde{u}_n^\pm + ik^*n_{0y}\tilde{v}_n^\pm = \frac{i\omega^*}{c_0^2}\tilde{p}_n^\pm \tag{14}$$

$$\left(\frac{d^2}{dz^2} - k_n^2 - k^{*2}n_{0y}^2 + \frac{i\omega^*}{v}\right) \begin{bmatrix} \tilde{u}_n^\pm \\ \tilde{v}_n^\pm \\ \tilde{w}_n^\pm \end{bmatrix} = \frac{\gamma}{\rho_0} \begin{bmatrix} ik_n \\ ik^*n_{0y} \\ d/dz \end{bmatrix} \tilde{p}_n^\pm \tag{15}$$

The solution of this system of equations is considered of the form

$$[\tilde{u}_n^\pm(z), \tilde{v}_n^\pm(z), \tilde{w}_n^\pm(z), \tilde{p}_n^\pm(z)] = e^{\alpha z} [U_n^\pm, V_n^\pm, W_n^\pm, P_n^\pm] \tag{16}$$

where  $\alpha$  is a constant to be determined. The substitution of (16) into (14) and (15) yields the homogeneous system of algebraic equations for the constants  $U_n^\pm, V_n^\pm, W_n^\pm, P_n^\pm$

$$\begin{bmatrix} \beta & 0 & 0 & ik_n \\ 0 & \beta & 0 & ik^*n_{0y} \\ 0 & 0 & \beta & \alpha \\ ik_n & ik^*n_{0y} & \alpha & i\omega_0^*/(\gamma c_0^2) \end{bmatrix} \cdot \begin{bmatrix} U_n^\pm \\ V_n^\pm \\ W_n^\pm \\ -\gamma P_n^\pm/\rho_0 \end{bmatrix} = 0 \tag{17}$$

where

$$\beta = \alpha^2 - k_n^2 - k^{*2}n_{0y}^2 + i\omega^*/v \tag{18}$$

Nontrivial solutions to Eq. (17) exist when the matrix is singular, which occurs when

$$\beta^2 \left( \beta - \frac{i\omega^*\gamma c_0^2}{v(\gamma c_0^2 - i\omega^*)} \right) = 0 \tag{19}$$

Eqs. (17) and (18) give

$$\alpha = \pm q_n \equiv \pm \sqrt{k_n^2 + k^{*2}n_{0y}^2 - i\omega^*/v}, \quad \text{Re}(q_n) \geq 0 \tag{20}$$

$$\alpha = \pm r_n \equiv \pm \sqrt{k_n^2 - k^{*2}(1 - n_{0y}^2)}, \quad \text{Re}(r_n) \geq 0, \quad n \neq 0 \tag{21}$$

To analyze the propagating and evanescent modes we consider the expression  $E_n$  under the square root in formula (21). Thus, this can be written as

$$E_n = \omega^2 [n^2 + 2nn_{0x}t - n_z^2 t^2]$$

where  $t = k^*/\omega = fT/c_0$ .

For  $n = 0$  there results  $E_0 = -k^{*2}n_{0z}^2$  and formula (21) yields

$$\alpha = \pm r_0 \equiv \pm ik^*n_{0z}$$

which describes a propagating mode. The mode corresponding to  $n = 1$  gives

$$E_1 = \omega^2 [1 + 2n_{0x}t - n_z^2 t^2]$$

In the domain of acoustical frequencies ( $f < 2$  kHz) and for spatial microfield variables ( $T \sim 10^{-6}$  m) there results  $t \sim 1$ . Since  $n_{0x}$  can be assumed positive we have  $E_1 > 0$  resulting that the first mode describes an

evanescent wave. Similarly, all the modes corresponding to  $n > 1$  give evanescent waves. In the case where the frequency is increasing (for example, in the ultrasonic domain) more and more modes turn into propagating modes. As we are mainly interested in audio frequency applications we consider the one propagating mode case; the general case can be treated similarly.

### 3.1. Representation formulas for the reduced problem

The velocity field corresponding to the incoming pressure wave can be written as

$$\mathbf{v}^0(x, z) = ik^* A \delta \mathbf{n}_0 e^{ik^*(n_{0x}x + n_{0z}z)} \quad (22)$$

Then, we have

$$p(x, z)/\rho_0 = \begin{cases} Ae^{ik^*(n_x x + n_z z)} + P_0^+ e^{ik^*(n_{0x}x - n_{0z}z)} + \sum_{n \neq 0} P_n^+ e^{ik_n x - r_n z}, & \text{in } \mathcal{D}^+ \\ P_0^- e^{ik^*(n_{0x}x + n_{0z}z)} + \sum_{n \neq 0} P_n^- e^{ik_n x + r_n z}, & \text{in } \mathcal{D}^- \end{cases} \quad (23)$$

Thus, the scattered pressure field consists of a superposition of an infinite number of wave modes. For audible frequencies only the lowest order mode is propagating; all the other modes describe standing waves which decay exponentially with the distance to the plane  $z = 0$ . In writing the solution representation (23) we considered also Sommerfeld's radiation condition. The constants  $P_n^\pm$  will be determined by using the boundary conditions. In the case of the viscous fluid these conditions are written by means of velocities. For the velocity field we obtain

$$\mathbf{v}(x, z) = \begin{cases} \mathbf{v}^0(x, z) + e^{ik^* n_{0x} x} \tilde{\mathbf{v}}_0^+(z) + \sum_{n \neq 0} e^{ik_n x} \tilde{\mathbf{v}}_n^+(z), & \text{in } \mathcal{D}^+ \\ e^{ik^* n_{0x} x} \tilde{\mathbf{v}}_0^-(z) + \sum_{n \neq 0} e^{ik_n x} \tilde{\mathbf{v}}_n^-(z), & \text{in } \mathcal{D}^- \end{cases} \quad (24)$$

where

$$\begin{aligned} \tilde{u}_0^\pm(z) &= U_0^\pm e^{\mp q_0 z} + ik^* n_{0x} \delta P_0^\pm / \rho_0 e^{\mp ik^* n_{0z} z} \\ \tilde{v}_0^\pm(z) &= V_0^\pm e^{\mp q_0 z} + ik^* n_{0y} \delta P_0^\pm / \rho_0 e^{\mp ik^* n_{0z} z} \end{aligned} \quad (25)$$

$$\begin{aligned} \tilde{w}_0^\pm(z) &= \pm \frac{ik^*}{q_0} [n_{0x} U_0^\pm + n_{0y} V_0^\pm] e^{\mp q_0 z} \mp ik^* n_{0z} \delta P_0^\pm / \rho_0 e^{\mp ik^* n_{0z} z} \\ \tilde{u}_n^\pm(z) &= U_n^\pm e^{\mp q_n z} + ik_n \delta P_n^\pm / \rho_0 e^{\mp r_n z} \\ \tilde{v}_n^\pm(z) &= V_n^\pm e^{\mp q_n z} + ik^* n_{0y} \delta P_n^\pm / \rho_0 e^{\mp r_n z} \\ \tilde{w}_n^\pm(z) &= \pm (ik_n U_n^\pm / q_n + ik^* n_{0y} V_n^\pm / q_n) e^{\mp q_n z} \mp r_n \delta P_n^\pm / \rho_0 e^{\mp r_n z} \end{aligned} \quad (26)$$

In writing the representation (24) of the velocity field we imposed again Sommerfeld's condition that apart from initial perturbation, the rest of the solution describes outgoing waves.

The velocity field is continuous across the plane  $z = 0$ . Indeed it is continuous along the gaps and it is vanishing on the strips. Consequently we can write

$$\mathbf{v}(x, \pm 0) \equiv e^{ik^* n_{0x} x} \tilde{\mathbf{v}}(x) = e^{ik^* n_{0x} x} \sum_{-\infty}^{+\infty} e^{i\alpha x} \tilde{\mathbf{v}}_n, \quad x \in \mathbb{R} \quad (27)$$

**Remark 1.** The Fourier coefficients  $\tilde{\mathbf{v}}_n = (\tilde{u}_n, \tilde{v}_n, \tilde{w}_n)$  are completely different from the functions  $\tilde{\mathbf{v}}_n^\pm(z) = (\tilde{u}_n^\pm(z), \tilde{v}_n^\pm(z), \tilde{w}_n^\pm(z))$ .



We express the unknown coefficients  $P_n^\pm, U_n^\pm, V_n^\pm$  in terms of the physical coefficients  $\tilde{v}_n$ . There results

$$P_0^+/\rho_0 = \frac{-ik^*n_{0x}\tilde{u}_0 - ik^*n_{0y}\tilde{v}_0 + q_0\tilde{w}_0 - i\delta k^*n_{0z}q_0 - \delta k^{*2}(1 - n_{0z}^2)}{\delta(k^{*2}n_{0x}^2 + k^{*2}n_{0y}^2 - ik^*n_{0z}q_0)}$$

$$P_0^-/\rho_0 = \frac{-ik^*n_{0x}\tilde{u}_0 - ik^*n_{0y}\tilde{v}_0 - q_0\tilde{w}_0}{\delta(k^{*2}n_{0x}^2 + k^{*2}n_{0y}^2 - ik^*n_{0z}q_0)} \tag{28}$$

$$U_0^+ = \tilde{u}_0 - ik^*n_{0x}\delta(1 + P_0^+/\rho_0), \quad U_0^- = \tilde{u}_0 - ik^*n_{0x}\delta P_0^-/\rho_0$$

$$V_0^+ = \tilde{v}_0 - ik^*n_{0y}\delta(1 + \varepsilon P_0^+/\rho_0), \quad V_0^- = \tilde{v}_0 - ik^*n_{0y}\delta P_0^-/\rho_0$$

$$P_n^\pm/\rho_0 = \frac{-ik_n\tilde{u}_n - ik^*n_{0y}\tilde{v}_n \pm q_n\tilde{w}_n}{\delta(k_n^2 + k^{*2}n_{0y}^2 - r_nq_n)}$$

$$U_n^\pm = \frac{(k^{*2}n_{0y}^2 - r_nq_n)\tilde{u}_n - k_nk^*n_{0y}\tilde{v}_n \mp ik_nq_n\tilde{w}_n}{k_n^2 + k^{*2}n_{0y}^2 - r_nq_n} \tag{29}$$

$$V_n^\pm = \frac{-k_nk^*n_{0y}\tilde{u}_n + (k_n^2 - r_nq_n)\tilde{v}_n \mp ik^*n_{0y}q_n\tilde{w}_n}{k_n^2 + k^{*2}n_{0y}^2 - r_nq_n}$$

The formulas (28) and (29) will be used for obtaining the integral equations for solving the problem.

#### 4. The integral equations of the problem

##### 4.1. The distributional equations

The periodic part of the velocity field was denoted in formulas 10 and 11 by  $\tilde{v}^\pm(x, z)$ . But for  $z = 0$  we have  $\tilde{v}^+(x, 0) = \tilde{v}^-(x, 0) = \tilde{v}(x)$ . To obtain the equations satisfied by the functions  $\tilde{u}(x), \tilde{v}(x), \tilde{w}(x)$  we impose the condition of continuity of the normal derivative of velocity along the aperture

$$\frac{\partial \tilde{v}^0(x, 0)}{\partial z} \exp\{-ik^*n_{0x}x\} + \frac{\partial \tilde{v}^+(x, 0)}{\partial z} = \frac{\partial \tilde{v}^-(x, 0)}{\partial z}, \quad x \in (-a, a)$$

There results the equations

$$\mathcal{P}_{(-a,a)} \left[ \sum_{n=-\infty}^{+\infty} (K_{11}^{(n)}\tilde{u}_n + K_{12}^{(n)}\tilde{v}_n) \exp\{in\omega x\} \right] = a d_1/T$$

$$\mathcal{P}_{(-a,a)} \left[ \sum_{n=-\infty}^{+\infty} (K_{21}^{(n)}\tilde{u}_n + K_{22}^{(n)}\tilde{v}_n) \exp\{in\omega x\} \right] = a d_2/T \tag{30}$$

$$\mathcal{P}_{(-a,a)} \left[ \sum_{n=-\infty}^{+\infty} K_3^{(n)}\tilde{w}_n \exp\{in\omega x\} \right] = a d_3/T$$

We add also the condition that on the screens the velocity vanishes. This may be expressed in the form

$$\mathcal{P}_{(a,b)}[\tilde{v}(x)] = 0 \tag{31}$$

Here we have denoted by  $\mathcal{P}_{(a,b)}$  the restriction operator on  $(a, b)$ . The coefficients entering into equations (30) have the expressions

$$\begin{aligned}
 K_{11}^{(0)} &= \frac{k^* n_{0z} \omega_0^* / v - k^{*2} n_{0y}^2 (q_0 - ik^* n_{0z})}{k^{*2} (n_{0x}^2 + n_{0y}^2) - ik^* q_0 n_{0z}}, & K_{12}^{(0)} &= \frac{k^{*2} n_{0x} n_{0y} (q_0 - ik^* n_{0z})}{k^{*2} (n_{0x}^2 + n_{0y}^2) - ik^* q_0 n_{0z}} \\
 K_{22}^{(0)} &= \frac{k^* n_{0z} \omega_0^* / v - k^{*2} n_{0x}^2 (q_0 - ik^* n_{0z})}{k^{*2} (n_{0x}^2 + n_{0y}^2) - ik^* q_0 n_{0z}}, & K_3^{(0)} &= \frac{q_0}{k^{*2} (n_{0x}^2 + n_{0y}^2) - ik^* q_0 n_{0z}} \\
 K_{11}^{(n)} &= \frac{-i\omega_0^* r_n / v + k^{*2} n_{0y}^2 (r_n - q_n)}{k_n^2 + k^{*2} n_{0y}^2 - r_n q_n}, & K_{12}^{(n)} &= \frac{-k_n k^* n_{0y} (r_n - q_n)}{k_n^2 + k^{*2} n_{0y}^2 - r_n q_n}, \quad n \neq 0 \\
 K_{22}^{(n)} &= \frac{-i\omega_0^* r_n / v + k_n^2 (r_n - q_n)}{k_n^2 + k^{*2} n_{0y}^2 - r_n q_n}, & K_3^{(n)} &= \frac{q_n}{k_n^2 + k^{*2} n_{0y}^2 - r_n q_n}, \quad K_{12}^{(n)} = K_{21}^{(n)} \\
 d_1 &= \frac{i\omega_0^* \delta k^* n_{0x} n_{0z} T / (av)}{k^* (n_{0x}^2 + n_{0y}^2) - iq_0 n_{0z}}, & d_2 &= \frac{n_{0y}}{n_{0x}} d_1, \quad d_3 = \frac{q_0 v}{\omega^* k^* n_{0x}} d_1
 \end{aligned}$$

By substituting the expressions of the Fourier coefficients (13) the relations (30) become

$$\int_{-a}^a [\tilde{u}(x') \mathbb{K}_{11}(x-x') + \tilde{v}(x') \mathbb{K}_{12}(x-x')] dx' = ad_1, \quad x \in (-a, a) \tag{32}$$

$$\int_{-a}^a [\tilde{u}(x') \mathbb{K}_{21}(x-x') + \tilde{v}(x') \mathbb{K}_{22}(x-x')] dx' = ad_2, \quad x \in (-a, a) \tag{33}$$

$$\int_{-a}^a \tilde{w}(x') \mathbb{K}_3(x-x') dx' = ad_3, \quad x \in (-a, a) \tag{34}$$

The kernels  $\mathbb{K}_{11} - \mathbb{K}_3$  have the expressions

$$\mathbb{K}_{mp}(x) = \sum_{n=-\infty}^{+\infty} K_{mp}^{(n)} \exp\{in\omega x\}, \quad \mathbb{K}_3(x) = \sum_{n=-\infty}^{+\infty} K_3^{(n)} \exp\{in\omega x\} \tag{35}$$

Thus, we obtained a system of two functional equations for determining the functions  $u(x)$  and  $v(x)$ , Eqs. (32) and (33), and an independent equation for the component  $w(x)$ , Eq. (34).

Eqs. (32)–(34) have resulted from the corresponding pseudodifferential equations. The Fourier series involved in the definition of the kernels  $\mathbb{K}_{11} - \mathbb{K}_3$  are not convergent as regular functions. They have to be considered as distributions [9,10].

#### 4.2. Reduction of the distributional equations to hypersingular integral equations

By using formulas (57)–(59) presented in the Appendix A, we can write the kernels  $\mathbb{K}_{11} - \mathbb{K}_3$  in the form

$$\mathbb{K}_{mp} = \mathbb{K}_{mp}^S + \mathbb{K}_{mp}^R$$

where the singular parts  $\mathbb{K}_{mp}^S$  and the regular parts  $\mathbb{K}_{mp}^R$  have the expressions

$$\begin{aligned} \mathbb{K}_{11}^S(x) &= b_{11} \left( -\frac{\omega}{2} \frac{1}{\sin^2(\omega x/2)} + ik^* n_{0x} \cot \frac{\omega x}{2} \right) + c_{11} \left( -\frac{2}{\omega} \log \left| 2 \sin \frac{\omega x}{2} \right| \right) \\ \mathbb{K}_{11}^R(x) &= K_{11}^{(0)} + \sum_{n \neq 0} \left[ K_{11}^{(n)} - b_{11} \left( |n\omega| + k^* n_{0x} \frac{n\omega}{|n\omega|} \right) - \frac{c_{11}}{|n\omega|} \right] \exp(in\omega x) \end{aligned} \quad (36)$$

$$\begin{aligned} \mathbb{K}_{12}^S(x) &= ib_{12} k^* n_{0y} \cot \frac{\omega x}{2} \\ \mathbb{K}_{12}^R(x) &= K_{12}^{(0)} + \sum_{n \neq 0} \left[ K_{12}^{(n)} - b_{12} k^* n_{0y} \frac{n\omega}{|n\omega|} \right] \exp(in\omega x) \end{aligned} \quad (37)$$

$$\begin{aligned} \mathbb{K}_{22}^S(x) &= b_{22} \left( -\frac{\omega}{2} \frac{1}{\sin^2(\omega x/2)} + ik^* n_{0x} \cot \frac{\omega x}{2} \right) + c_{22} \left( -\frac{2}{\omega} \log \left| 2 \sin \frac{\omega x}{2} \right| \right) \\ \mathbb{K}_{22}^R(x) &= K_{22}^{(0)} + \sum_{n \neq 0} \left[ K_{22}^{(n)} - b_{22} \left( |n\omega| + k^* n_x \frac{n\omega}{|n\omega|} \right) - \frac{c_{22}}{|n\omega|} \right] \exp(in\omega x) \end{aligned} \quad (38)$$

$$\begin{aligned} \mathbb{K}_3^S(x) &= b_3 \left( -\frac{\omega}{2} \frac{1}{\sin^2(\omega x/2)} + ik^* n_{0x} \cot \frac{\omega x}{2} \right) + \frac{2b_3}{\omega} \log \left| 2 \sin \frac{\omega x}{2} \right| \\ \mathbb{K}_3^R(x) &= K_3^{(0)} + \sum_{n \neq 0} \left[ K_3^{(n)} + b_3 \left( |n\omega| + k^* n_{0x} \frac{n\omega}{|n\omega|} \right) - \frac{c_3}{|n\omega|} \right] \exp(in\omega x) \end{aligned} \quad (39)$$

The constants entering in these relationships have the expressions

$$\begin{aligned} b_{11} &= \frac{-2i\omega^*/v}{k^{*2} + i\omega^*/v}, \quad b_{12} = \frac{k^{*2} - i\omega^*/v}{k^{*2} + i\omega^*/v}, \quad b_{22} = -1, \quad b_3 = \frac{2}{k^{*2} + i\omega^*/v} \\ c_{11} &= \frac{-2k^{*4} n_{0y}^2 i\omega^*/v + (i\omega^*/v)^3 + k^{*4} (-2k^{*2} n_{0y}^2 + 3i\omega^*/v)}{2(k^{*2} + i\omega^*/v)^2} \\ c_{22} &= \frac{i\omega^*/v(i\omega^*/v - 3k^{*2} n_{0y}^2) + k^{*2}(i\omega^*/v + k^{*2} n_{0y}^2)}{2(k^{*2} + i\omega^*/v)} \\ c_3 &= \frac{-k^{*4} + 2k^{*4} n_{0y}^2 + (2k^{*2} n_{0y}^2 - 3i\omega^*/v)i\omega^*/v}{2(k^{*2} + i\omega^*/v)^2} \end{aligned}$$

The general terms in brackets in expressions (36), (37) and (39) decrease for  $n \rightarrow \infty$  like  $|n|^{-2}$  and, consequently, the functions  $\mathbb{K}_{mp}^R(x)$  are continuous on the interval  $[-a, a]$ . Finally, there results the following hyper-singular integral equations for solving the problem

$$\begin{aligned} b_{11} \left( -\frac{\omega}{2} \int_{-a}^a \frac{\tilde{u}(x') dx'}{\sin^2(\omega(x-x')/2)} + ik^* n_{0x} \int_{-a}^a \tilde{u}(x') \cot \frac{\omega(x-x')}{2} dx' \right) \\ - \frac{2c_{11}}{\omega} \int_{-a}^a \tilde{u}(x') \log \left| 2 \sin \frac{\omega(x-x')}{2} \right| dx' + b_{12} ik^* n_{0y} \int_{-a}^a \tilde{v}(x') \cot \frac{\omega(x-x')}{2} dx' \\ + \int_{-a}^a [\tilde{u}(x') \mathbb{K}_{11}^R(x-x') + \tilde{v}(x') \mathbb{K}_{12}^R(x-x')] dx' = ad_1 \end{aligned} \quad (40)$$

$$\begin{aligned} b_{12} ik^* n_{0y} \int_{-a}^a \tilde{u}(x') \cot \frac{\omega(x-x')}{2} dx' - b_{22} \frac{\omega}{2} \int_{-a}^a \frac{\tilde{v}(x') dx'}{\sin^2(\omega(x-x')/2)} \\ + b_{22} ik^* n_{0x} \int_{-a}^a \tilde{v}(x') \cot \frac{\omega(x-x')}{2} dx' - \frac{2c_{22}}{\omega} \int_{-a}^a \tilde{v}(x') \log \left| 2 \sin \frac{\omega(x-x')}{2} \right| dx' \\ + \int_{-a}^a [\tilde{u}(x') \mathbb{K}_{21}^R(x-x') + \tilde{v}(x') \mathbb{K}_{22}^R(x-x')] dx' = ad_2 \end{aligned} \quad (41)$$

$$\begin{aligned} b_3 \left( -\frac{\omega}{2} \int_{-a}^a \frac{\tilde{w}(x') dx'}{\sin^2(\omega(x-x')/2)} + ik^* n_{0x} \int_{-a}^a \tilde{w}(x') \cot \frac{\omega(x-x')}{2} dx' \right) \\ - \frac{2c_3}{\omega} \int_{-a}^a \tilde{w}(x') \log \left| 2 \sin \frac{\omega(x-x')}{2} \right| dx' + \int_{-a}^a \tilde{w}(x') \mathbb{K}_3^R(x-x') dx' = ad \end{aligned} \quad (42)$$

It is to be noticed that we obtained a system of hypersingular integral Eqs. (40) and (41) for determining the components of velocity in the  $z = 0$  plane and another hypersingular integral equation for determining the velocity component in the  $z$ -direction. As the solutions satisfy the conditions  $\tilde{u}(\pm a) = \tilde{v}(\pm a) = \tilde{w}(\pm a) = 0$  the equations can be transformed into integro-differential equations with a Cauchy-type singularity.

The hypersingular integral equation is the natural mathematical tool for approaching the diffraction of damped acoustical waves. Direct numerical methods for solving hypersingular integral equations were developed by Kutt [11] and Dragos [12].

### 5. Reduction of the integral equations to infinite systems of algebraic equations

Instead of using collocation type methods for approximating the solution of the singular integral Eqs. (40)–(42) we prefer a Galerkin-type approach based on a special basis of the corresponding Hilbert space. The method takes advantage of the form of integral equations by using some spectral-type relationships, Fast Fourier Transform of some smooth function and summation of rapidly convergent infinite series.

#### 5.1. Galerkin's method for solving the integral equations

In the space  $\mathcal{H}^{1/2}(-a, a)$  of functions continuous on  $[-a, a]$  with derivatives having singularities of order  $1/2$  at extremities we consider the basis

$$\left\{ \sin \left( n \arccos \frac{x}{a} \right) \right\}, \quad n = 1, 2, \dots$$

We represent the function  $\tilde{\mathbf{v}}(x)$  as

$$\begin{bmatrix} \tilde{u}(x) \\ \tilde{v}(x) \\ \tilde{w}(x) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} \alpha_m \\ \beta_m \\ \gamma_m \end{bmatrix} \sin \left( m \arccos \frac{x}{a} \right) \tag{43}$$

By applying the *Galerkin's method* to the singular integral Eqs. (40)–(42) we obtain the following sets of infinite linear equations for determining the constants  $\alpha_m, \beta_m, \gamma_m$

$$\sum_{m=1}^{\infty} A_{pm}^{(1)} \alpha_m + \sum_{m=1}^{\infty} B_{pm}^{(1)} \beta_m = \frac{d_1}{2\pi} \delta_{p,1}, \quad p = 1, 2, \dots \tag{44}$$

$$\sum_{m=1}^{\infty} A_{pm}^{(2)} \alpha_m + \sum_{m=1}^{\infty} B_{pm}^{(2)} \beta_m = \frac{d_2}{2\pi} \delta_{p,1}, \quad p = 1, 2, \dots$$

$$\sum_{m=1}^{\infty} C_{pm} \gamma_m = \frac{d_3}{2\pi} \delta_{p,1}, \quad p = 1, 2, \dots \tag{45}$$

The coefficients  $A_{pm}^{(i)}, B_{pm}^{(i)}, C_{pm}$  have the expressions

$$\begin{aligned} A_{pm}^{(1)} &= -\frac{b_{11}\omega}{2} \mathbb{I}_{pm}^{(2)} + b_{11}ik^* n_{0x} \mathbb{I}_{pm}^{(1)} - \frac{2c_{11}}{\omega} \mathbb{I}_{pm}^{(0)} + A_{pm}^{1R} \\ B_{pm}^{(1)} &= b_{12}ik^* n_{0y} \mathbb{I}_{pm}^{(1)} + B_{pm}^{1R}, \quad A_{pm}^{(2)} = b_{12}ik^* n_y \mathbb{I}_{pm}^{(1)} + A_{pm}^{2R} \end{aligned} \tag{46}$$

$$B_{pm}^{(2)} = -b_{22} \frac{\omega}{2} \mathbb{I}_{pm}^{(2)} + b_{22}ik^* n_{0x} \mathbb{I}_{pm}^{(1)} - \frac{2c_{22}}{\omega} \mathbb{I}_{pm}^{(0)} + B_{pm}^{2R}$$

$$C_{pm} = -\frac{b_3\omega}{2} \mathbb{I}_{pm}^{(2)} + b_3ik^* n_{0x} \mathbb{I}_{pm}^{(1)} - \frac{2c_3}{\omega} \mathbb{I}_{pm}^{(0)} + C_{pm}^R \tag{47}$$

The integrals  $\mathbb{I}_{pm}^{(i)}$  are given in the Appendix A and the “regular parts”  $A_{pm}^{1R} - C_{pm}^R$  can be written as

$$A_{pm}^{1R} = \frac{K_{11}^{(0)}}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \cdot \left[ K_{11}^{(n)} + (-1)^{p+m} K_{11}^{(-n)} - \left( b_{11} n \omega + \frac{c_{11}}{n\omega} \right) (1 + (-1)^{p+m}) - b_{11} k^* n_{0x} (1 - (-1)^{p+m}) \right] \quad (48)$$

$$A_{pm}^{2R} = B_{pm}^{1R} = \frac{K_{12}^{(0)}}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \cdot \left[ K_{12}^{(n)} + (-1)^{p+m} K_{12}^{(-n)} - b_{12} k^* n_{0y} (1 - (-1)^{p+m}) \right] \quad (49)$$

$$B_{pm}^{2R} = \frac{K_{22}^{(0)}}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \cdot \left[ K_{22}^{(n)} + (-1)^{p+m} K_{22}^{(-n)} - \left( b_{22} n \omega + \frac{c_{22}}{n\omega} \right) (1 + (-1)^{p+m}) - b_{22} k^* n_{0x} (1 - (-1)^{p+m}) \right] \quad (50)$$

$$C_{pm}^R = \frac{K_3^{(0)}}{4} \delta_{p,1} \delta_{m,1} + i^{p-m} m p \sum_{n=1}^{\infty} \frac{J_p(an\omega) J_m(an\omega)}{(an\omega)^2} \cdot \left[ K_3^{(n)} + (-1)^{p+m} K_3^{(-n)} - \left( b_3 n \omega + \frac{c_3}{n\omega} \right) (1 + (-1)^{p+m}) - c_3 k^* n_{0x} (1 - (-1)^{p+m}) \right] \quad (51)$$

### 5.2. Numerical realization of the method

The systems of linear Eqs. (44) and (45) have good properties from a computational point of view. Thus, the integrals  $\mathbb{I}_{pm}^{(j)}$  are given by explicit relationships (64)–(66) in the Appendix A. The double integrals involved in their expressions can be written as 2-D cosine Fourier transforms. The integrands in  $\mathbb{I}_{pm}^{(j)}$  are smooth functions such that these integrals can be computed efficiently by using the 2-D discrete cosine transform function of MATLAB. Finally, the coefficients  $A_{pm}^{1R} - C_{pm}^R$  can be obtained directly by summing the infinite series in formulas (48)–(51). By subtracting several terms, and summing them separately, we have transformed the initial infinite series into rapidly convergent series.

By the approach used in the previous section we have transformed the convolution operators in the Fourier transform domain; this transform converts the convolution to a product “diagonalizing” the operators. This is why the finite sections of the resulting infinite systems of linear equations have relatively low condition numbers and the series giving the functions  $\tilde{u}(x)$ ,  $\tilde{v}(x)$ ,  $\tilde{w}(x)$  are rapidly convergent.

## 6. Transmission coefficient: Numerical results

In order to determine how much of the incoming plane wave is passing through the grating we will consider the transmission coefficient  $\tau$  defined in Mechels’ book [13] (p. 432) as the squared magnitude of the ratio of transmitted to incident

$$\tau = |P_0^- / A|^2 \quad (52)$$

pressures. Thus, once the solutions of the systems (44) and (45) are determined the functions  $\tilde{u}(x)$ ,  $\tilde{w}(x)$  can be introduced in formula (43) for obtaining the corresponding Fourier coefficients. Finally, formulas (28) and (52) provide the value of the transmission coefficient as

$$\tau = \left| \frac{q_0 w_1 + i k^* u_1 \sin \theta_0}{i q_0 \cos \theta_0 + k^* \sin^2 \theta_0} \frac{a \pi}{2(a + b) A \delta k^*} \right|^2 \quad (53)$$

For determining the influence of air viscosity on the transmission coefficient in Fig. 2a we have plotted  $|\tau|$  versus slits width  $d$  for periodic spacing values  $w = 25, 50, 100, 200 \mu\text{m}$ , frequency = 20 kHz, and the unit vector of the incoming wave  $\mathbf{n}_0 = (\sqrt{2}/2, 0, -\sqrt{2}/2)$ , in the nonviscous case, obtained also by means of a Galerkin

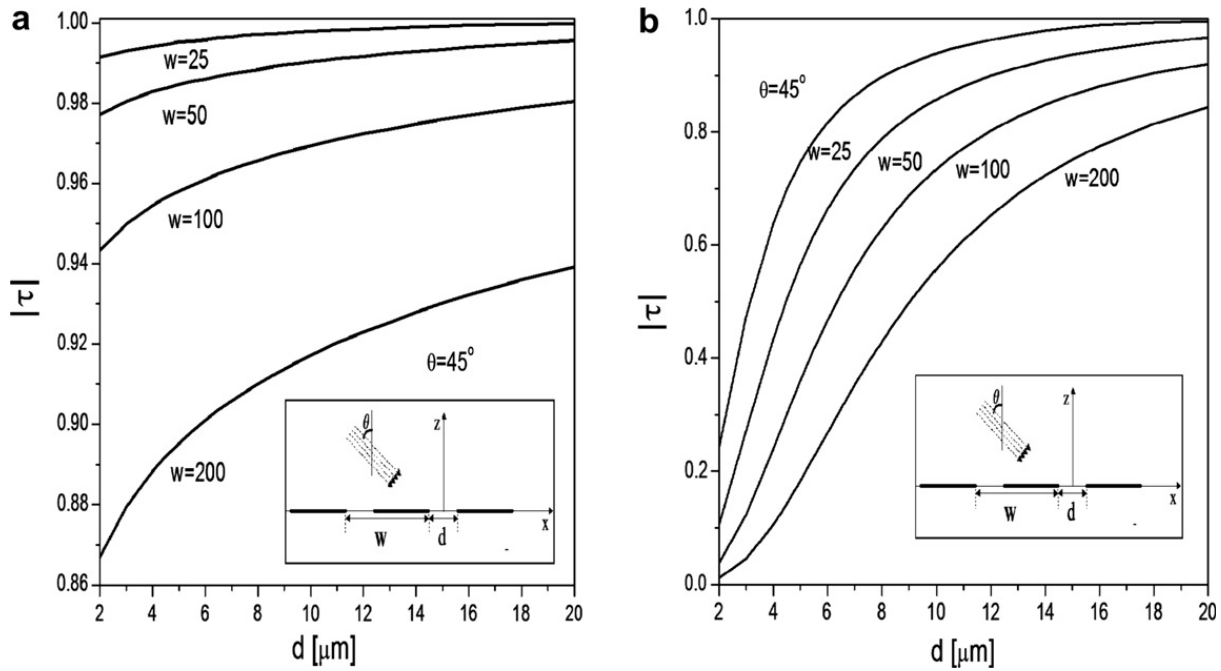


Fig. 2. The transmission coefficient  $\tau$  versus slit's width  $d$  for periodic spacing. (a) the nonviscous case, (b) the viscous case.

approach. In Fig. 2b there is plotted the same transmission coefficient in the case where the effect of viscosity of air is included. For small slits of width  $d$  the influence of viscosity on the transmission coefficient is significant. Hence, in order to avoid the excessive attenuation of the sound due to the grating the openings in the periodic screen have to be of order of 10  $\mu\text{m}$ .

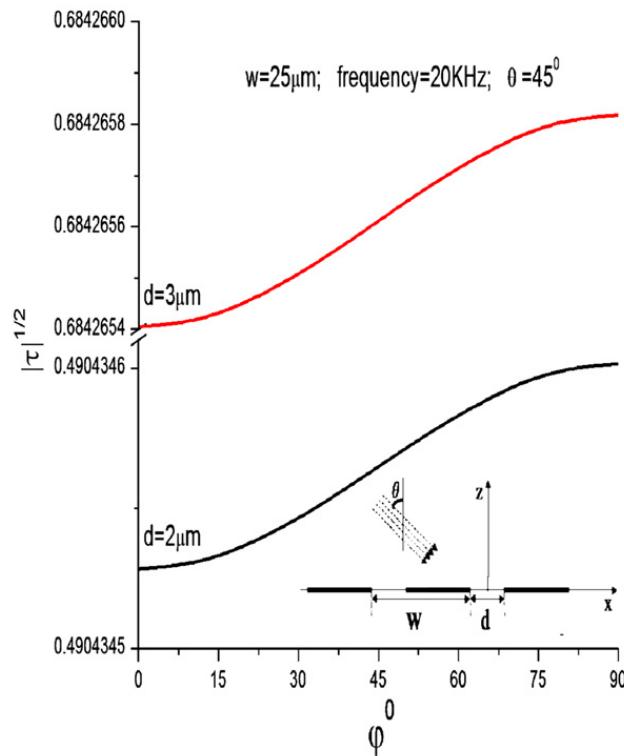


Fig. 3. The dependence of transmission coefficient  $\tau$  versus azimuth coordinate  $\varphi$ .

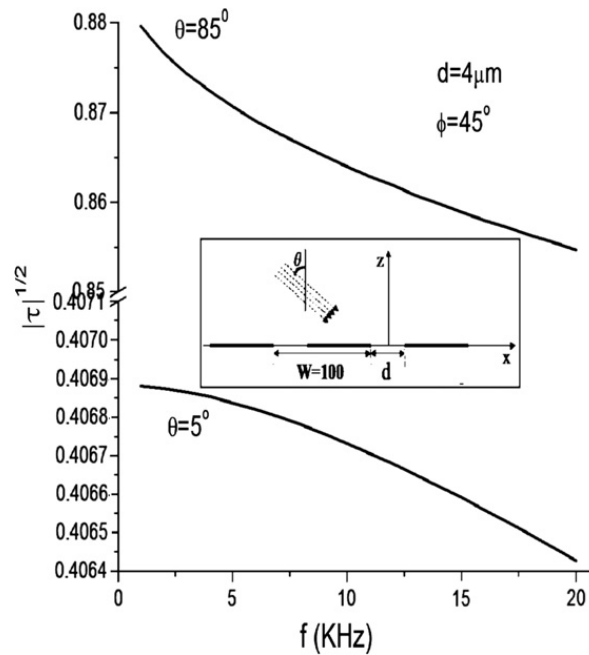


Fig. 4. The dependence of transmission coefficient  $\tau$  versus frequency.

We analyze now the dependence of the transmission coefficient upon the horizontal direction of the incoming plane wave. For this we considered a spherical system of coordinates  $x = r \sin \theta \cos \varphi$ ,  $y = r \sin \theta \sin \varphi$ ,  $z = r \cos \theta$ .

In Fig. 3 there are plotted the values of  $|\tau|^{1/2}$  versus azimuth coordinate  $\varphi$  for constant colatitudes coordinate  $\theta = \pi/4$ , and two values: 2 and 3  $\mu\text{m}$  of the slit width. It is to be noticed that, practically, the transmission coefficient does not depend on  $\varphi$ .

Finally, Fig. 4 plots the dependence of the transmission coefficient with frequency of the incoming wave. For each value of  $\theta$  the transmission coefficient shows a weak dependence on frequency in the audible frequency range.

### Acknowledgement

This work has been supported through NIH Grant R01 DC05762-1A1 to R.N.M.

### Appendix A. A.1. Some Fourier series

Starting with the equality [14]

$$\sum_{n=1}^{\infty} \frac{\cos(n\omega x)}{n\omega} = -\frac{1}{\omega} \log \left| 2 \sin \frac{\omega x}{2} \right| \quad (54)$$

we can write

$$\sum_{n=1}^{\infty} \sin(n\omega x) = \frac{1}{2} \cot \frac{\omega x}{2} \quad (55)$$

$$\sum_{n=1}^{\infty} n\omega \cos(n\omega x) = -\frac{\omega}{4} \frac{1}{\sin^2(\omega x/2)} \quad (56)$$

the distributions in the right-hand side being considered as regularizations of the corresponding functions [15,16,17].

*A.2. Integrals used in Section 4*

We start with formula [18]

$$-\frac{1}{\pi} \int_{-1}^1 \frac{\cos(n \arccos x')}{\sqrt{1-x'^2}} \log|x-x'| dx' = \begin{cases} \cos(n \arccos x)/n, & n \geq 1 \\ \log 2, & n = 1 \end{cases} \tag{57}$$

By using this relationship we can write also

$$-\frac{1}{\pi} \int_{-1}^1 \sin(n \arccos x') \log|x-x'| dx' = \begin{cases} \frac{\cos(n-1)\theta}{2(n-1)} - \frac{\cos(n+1)\theta}{2(n+1)}, & n > 1 \\ \frac{\log 2}{2} - \frac{\cos(2\theta)}{4}, & n = 1 \end{cases} \tag{58}$$

where  $\theta = \arccos x$ . The derivatives of relation (58) give

$$\frac{1}{\pi} \int_{-1}^1 \frac{\sin(n \arccos x')}{x-x'} dx' = \cos(n\theta), \quad n \geq 1 \tag{59}$$

$$-\frac{1}{\pi} \int_{-1}^1 \frac{\sin(n \arccos x')}{(x-x')^2} dx' = n \frac{\sin(n\theta)}{\sin \theta} \tag{60}$$

The change of integration variable  $x' = \cos \theta'$  in the formulas (58)–(60) gives

$$-\frac{1}{\pi} \int_0^\pi \sin(n\theta') \sin \theta' \log|\cos \theta - \cos \theta'| d\theta' = \begin{cases} \frac{\cos(n-1)\theta}{2(n-1)} - \frac{\cos(n+1)\theta}{2(n+1)}, & n > 1 \\ \frac{\log 2}{2} - \frac{\cos(2\theta)}{4}, & n = 1 \end{cases} \tag{61}$$

$$\frac{1}{\pi} \int_0^\pi \frac{\sin(n\theta') \sin \theta'}{\cos \theta - \cos \theta'} d\theta' = \cos(n\theta), \quad n \geq 1 \tag{62}$$

$$-\frac{1}{\pi} \int_0^\pi \frac{\sin(n\theta') \sin \theta'}{(\cos \theta - \cos \theta')^2} d\theta' = n \frac{\sin(n\theta)}{\sin \theta}, \quad n \geq 1 \tag{63}$$

By using formulas (61)–(63) there results

$$\mathbb{I}_{pm}^{(2)} \equiv \frac{1}{a^2 \pi^2} \int_{-a}^a \int_{-a}^a \frac{\sin(m\theta') \sin(p\theta)}{\sin^2(\omega(x-x')/2)} dx' dx \tag{64}$$

$$= -\frac{2m}{a^2 \omega^2} \delta_{p,m} + \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \left[ \frac{1}{\sin^2 Z} - \frac{1}{Z^2} \right] dS_{pm}$$

$$\mathbb{I}_{pm}^{(1)} \equiv \frac{1}{a^2 \pi^2} \int_{-a}^a \int_{-a}^a \sin(m\theta') \sin(p\theta) \cot \frac{\omega(x-x')}{2} dx' dx \tag{65}$$

$$= \frac{1}{2a\omega} (\delta_{m+1,p} - \delta_{m,p+1}) + \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \left[ \cot Z - \frac{1}{Z} \right] dS_{pm}$$

$$\mathbb{I}_{pm}^{(0)} \equiv \frac{1}{a^2 \pi^2} \int_{-a}^a \int_{-a}^a \sin(m\theta') \sin(p\theta) \log \left| 2 \sin \frac{\omega(x-x')}{2} \right| dx' dx \tag{66}$$

$$= \begin{cases} .125[(\delta_{p+2,m} - \delta_{p,m})/(m-1) + (\delta_{p,m+2} - \delta_{p,m})/(m+1)], & m > 1 \\ .25 \log(\omega a/2) \delta_{p1} \delta_{m1} + 0.0625(\delta_{p,3} - \delta_{p,1}) \delta_{m,1}, & m = 1 \end{cases}$$

$$+ \frac{1}{\pi^2} \int_0^\pi \int_0^\pi \log \left| \frac{\sin Z}{Z} \right| dS_{pm}$$

Here we have denoted

$$\theta' = \arccos(x'/a), \quad \theta = \arccos(x/a)$$

$$Z = \frac{\omega a}{2} (\cos \theta - \cos \theta')$$

$$dS_{pm} = \sin(m\theta') \sin(p\theta) \sin \theta' \sin \theta \, d\theta \, d\theta'$$



### A.3. A Fourier Transform

We consider the generating function of Bessel's functions

$$\exp\left\{\frac{z}{2}\left(w - \frac{1}{w}\right)\right\} = \sum_{n=-\infty}^{\infty} J_n(z)w^n$$

and take  $w = i \exp\{i\varphi\}$ . There results

$$\exp\{iz \cos \varphi\} = \sum_{n=-\infty}^{\infty} J_n(z)i^n \exp\{in\varphi\}$$

By using the orthogonality relationship of the complex exponential there results

$$\int_0^\pi \exp\{iz \cos \varphi\} \cos(m\varphi) d\varphi = \pi i^m J_m(z) \tag{67}$$

Then, we have

$$\int_{-a}^a \sin\left(p \arccos \frac{x}{a}\right) \exp\{in\omega x\} dx = a \int_0^\pi \sin(p\theta) \sin \theta \exp\{in\omega \cos \theta\} d\theta$$

and using the Eq. (67) and some properties of Bessel's functions we finally obtain formula

$$\int_{-a}^a \sin\left(p \arccos \frac{x}{a}\right) \exp\{in\omega x\} dx = \pi i^{p-1} \frac{p}{n\omega} J_p(an\omega). \tag{68}$$

### References

- [1] H. Lamb, *Hydrodynamics*, Cambridge University Press, Cambridge, 1932.
- [2] J.W. Miles, On Rayleigh scattering by a grating, *Wave Motion* 4 (1982) 285–292.
- [3] J.D. Achenbach, Z.L. Li, Reflexion and transmission of scalar waves by a periodic array of screens, *Wave Motion* 8 (1986) 225–234.
- [4] E. Scarpetta, M.A. Sumbatyan, Explicit analytical results for one-mode oblique penetration into a periodic array of screens, *IMA Journal of Applied Mathematics* 56 (1996) 109–120.
- [5] J.B. Alblas, On the diffraction of sound waves in a viscous medium, *Applied Scientific Research A6* (1957) 237–262.
- [6] A.D. Pierce, *Acoustics*, Mc Graw-Hill, New York, 1981.
- [7] D. Homentcovschi, R.N. Miles, L. Tan, Influence of viscosity on the diffraction of sound by a periodic array of screens, *Journal of Acoustical Society of America* 117 (5) (2005) 2761–2771.
- [8] P.M. Morse, K.U. Ingard, *Theoretical Acoustics*, Princeton University Press, New Jersey, 1968.
- [9] V.S. Vladimirov, *Equations of Mathematical Physics*, Marcel Dekker Inc., New York, 1971.
- [10] V.S. Vladimirov, *Methods of the Theory of Generalized Functions*, Taylor & Francis, London and New York, 2002.
- [11] H.R. Kutt, The numerical evaluation of principal value integrals by finite part integration, *Numerische Mathematik* 24 (1975) 205–210.
- [12] L. Dragos, Integration of Prandtl's equation with the aid of quadrature formulae of Gauss type, *Quarterly of Applied Mathematics* 52 (1994) 23–29.
- [13] F.P. Mechel, *Formulas of Acoustics*, Springer-Verlag, Berlin, Heidelberg, New York, 2002.
- [14] I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, 1994.
- [15] D.S. Jones, *The Theory of Generalized Functions*, Cambridge University Press, Cambridge, 1982.
- [16] A.H. Zemanian, *Distribution Theory and Transform Analysis*, Mc.Graw-Hill, 1965.
- [17] J.C. Ferreira, *Introduction to the Theory of Distributions*, Addison-Wesley Longman, 1997.
- [18] G.M.L. Gladwell, *Contact problems in Classical Theory of Elasticity*, Alphen aan Rijn, Sijthoff and Noordhoff, 1980.